Geometry Key Concept
College- and Career-Ready Standards for Mathematics with Clarifying Comments

Kindergarten – Fifth Grade

The purpose of this document is to provide clarifying comments that will assist educators in knowing specifically what geometric shapes, attributes and/or properties a student should work with, know and understand at a particular grade. The clarifying comments are italicized and are not part of the original College- and Career-Ready Standards for Mathematics. When working with the Geometry Key Concept attention should also be paid to the standards set forth in the Measurement Key Concept, most of which are not addressed in this document. The standards are listed in numerical order as they appear in the standards document, not in a teaching sequence.

Kindergarten

Action Verbs: Identify, describe, classify and explain as 2-D or 3-D, analyze, compare, draw

Shapes: Two-Dimensional: triangle, square, rectangle, hexagon, circle,
Three-Dimensional: cone, cube, cylinder and sphere

New Vocabulary: Two-Dimensional: triangle, square, rectangle, hexagon, circle;
Three-Dimensional: cone, cube, cylinder and sphere;
Location: below, above, beside, between, inside, outside, in front of, or behind;
model

Prior Knowledge: While students may have had some informal exposure to shapes in everyday situations, this may be their first formal introduction to 2-D and 3-D shapes.

Focus of Learning for Kindergarten: The emphasis for 2-D and 3-D geometry in kindergarten is on identifying and describing shapes in everyday situations. Students should be able to classify the shapes listed above under "New Vocabulary" as either 2-D or 3-D and explain the basis for their thinking. In order to classify shapes, students need opportunities to analyze and compare 2-D and 3-D shapes in different sizes and orientations using informal language. The purpose is so that students begin to identify attributes which they will deal with in 1st grade. To help students begin to relate to the attributes of 2-D versus 3-D shapes, they should be given opportunities to draw 2-D shapes and create models of 3-D shapes.

K.G.1 - Describe positions of objects by appropriately using terms, including *below, above, beside, between, inside, outside, in front of,* or *behind.*

K.G.2 - Identify and describe a given shape and shapes of objects in everyday situations to include two-dimensional shapes (i.e., triangle, square, rectangle, hexagon, and circle) and three-dimensional shapes (i.e., cone, cube, cylinder, and sphere).

K.G.3 - Classify shapes as two-dimensional/flat or three-dimensional/solid and explain the reasoning used.

Three-Dimensional Defining Attributes

- *Length, width, height = Three-dimensional*
- *Understand that 3-D shapes (polyhedra/polyhedron) have flat faces. Students do not need to know the terms Polyhedra/polyhedron – that is teacher knowledge.*

Two-Dimensional
Length and width or length and height but not length, width and height

K.G.4 - Analyze and compare two- and three-dimensional shapes of different sizes and orientations using informal language.

K.G.5 - Draw two-dimensional shapes (i.e., square, rectangle, triangle, hexagon, and circle) and create models of three-dimensional shapes (i.e., cone, cube, cylinder, and sphere).

1st Grade

Action Verbs: Identify, name, distinguish between defining and non-defining attributes, combine, partition

Shapes: Two-Dimensional: square, rectangle, triangle, hexagon, rhombus, trapezoid, and circle
Three-Dimensional: cube, rectangular prism, cone and cylinder

New Vocabulary: defining attributes, non-defining attributes, hexagon, rhombus, trapezoid, rectangular prism, partition

Prior Knowledge: In K the main focus was on identifying 2-D versus 3-D shapes.

Focus of Learning for 1st Grade: Students in 1st grade are working with some of the same shapes as in K (triangle, square, rectangle, hexagon, circle, cone, cube, cylinder and sphere). However, they are adding the rhombus and trapezoid. The K shapes are repeated and additional shapes added because the emphasis in 1st grade is on distinguishing between defining and non-defining attributes. To help students focus on defining versus non-defining attributes, students should be given the opportunity to partition 2-D shapes into two or four equal parts and to combine 2-D and 3-D shapes to form a composite shape. Again, the goal for partitioning and combining is to encourage students to focus on the shapes' attributes. In addition, it is important that students recognize the equality of parts when partitioning shapes.

1.G.1 - Distinguish between a two-dimensional shape's defining (e.g., number of sides) and non-defining attributes (e.g., color).

Two-Dimensional Defining Attributes for 1st grade

- *Closed figure*
- *Straight sides/edges*
- *Number of sides/edges*
- *Number of corners/vertices*
- *Relationship between length of edges (all equal edges; all unequal edges; some equal and some unequal edges)*

Circle Defining Attributes for 1st grade

- *No corner/vertex(vertices)*

1.G.2 - Combine two-dimensional shapes (i.e., square, rectangle, triangle, hexagon, rhombus, and trapezoid) or three-dimensional shapes (i.e., cube, rectangular prism, cone, and cylinder) in more than one way to form a composite shape.

- *This means to put together 2-D to form different 2-D or put together 3-D to form different 3-D shapes without naming the new shape but simply focusing on what shapes were combined, what were the defining attributes of the original compared to the defining attributes of the new shape - the defining attributes listed under 1.G.1 above.*

1.G.3 - Partition two-dimensional shapes (i.e., square, rectangle, circle) into two or four equal parts.

- *Note that this is the beginning development of the concept of fractions. The concepts of halves or fourths do not appear in any other strands in this grade level. So this standard deserves special*

attention since it is the first exposure to fractions. The emphasis here is on the idea of equal shares/parts. The formal idea and symbolism of fractions are introduced in grade 3.

1.G.4 - Identify and name two-dimensional shapes (i.e., square, rectangle, triangle, hexagon, rhombus, trapezoid, and circle).

2nd Grade

Action Verbs: Recognize, identify, partition, and draw

Shapes: Two-Dimensional: square, rectangle, triangle, hexagon, circle
Three-Dimensional: cube
Classifications: Quadrilaterals

New Vocabulary: quadrilaterals, angles, faces, halves, fourths, a half of, and a fourth of.

Prior Knowledge: In prior grades students focused on identifying shapes and a beginning understanding of defining (number of sides) and non-defining (color) attributes. In addition, students partitioned 2-D shapes into two or four equal parts. The emphasis in 1st grade was on equality of parts. Students build on that understanding in 2nd grade.

Focus of Learning for 2nd Grade: In 2nd grade students use their knowledge of attributes such as number of sides to recognize and draw shapes when the teacher specifies attributes such as number of angles or number of equal faces. Giving attributes and allowing students to draw will enable them to see that given attributes can apply to more than one shape. Allow students to share drawings and discuss how the given attributes are represented in the drawing. This sets the stage for 3rd grade when students begin to classify shapes. Also, students build on their 1st grade knowledge by not only partitioning squares, rectangles and circles into two or four equal parts but they must also describe the parts using the words *halves, fourths, a half of,* and *a fourth of.* It is extremely important that when partitioning shapes of equal size/shape, students begin to recognize and verbalize that as the number of parts increases, the size of the parts decreases (becomes smaller) – the more parts a shape is divided into, the smaller each part.

2.G.1 - Identify triangles, quadrilaterals, hexagons, and cubes. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.
***NOTE**: Students should recognize the 2-D and 3-D shapes specified based on specified attributes. However, care should be exercised at this grade level when requiring students to draw shapes having specified attributes. For example, 2nd grade students should not be given the number of angles in a cube and then expect a cube to be draw. Rather, when specifying a given number of angles, student drawings should be limited to the specified 2-D shapes and when specifying a given number of equal faces, the only 3-D shape students should be required to draw is the cube. The appropriate "specified attributes" to which students should have been exposed in previous grades and will be exposed during this grade are listed below. Giving attributes and allowing students to draw will enable them to see that given attributes can apply to more than one shape. Allow students to share drawings and discuss how the given attributes are represented in the drawing. This sets the stage for 3rd grade when students begin to classify shapes.*

Two-Dimensional Defining Attributes for 2nd grade

- *Closed figure (discussed in 1st grade with other shapes)*
- *Straight sides\edges (discussed in 1st grade with other shapes)*
- *Number of sides/edges (discussed in 1st grade with other shapes)*
- *Number of angles – angle is a new term and should be informally introduced. Types of angles will be formally addressed in 3rd grade. In 1st grade it was introduced as number of corners/vertices*

- *Relationship between length of sides/edges (equal, all unequal or one pair equal, one pair unequal) of edges (length is compared directly or visually, not compared by measuring).*
- *Equal faces*
- *Length and width = Two-dimensional*

2.G.2 - Partition a rectangle into rows and columns of same-size squares to form an array and count to find the total number of parts.

- *Note: This should be related to standard 2.ATO.4 which requires students to make an array and discuss as repeated addition. This lays the foundation for the introduction of the concepts of multiplication (3.ATO.3) and area (3.MD.5&6) in grade 3.*

2.G.3 - Partition squares, rectangles and circles into two or four equal parts, and describe the parts using the words *halves, fourths, a half of,* and *a fourth of.* Understand that when partitioning a square, rectangle or circle into two or four equal parts, the parts become smaller as the number of parts increases.

- *Note that informally students were introduced to the concept of fractions at 1.G.3. This builds on that informal introduction. So this standard deserves special attention. Emphasis here is on the number of parts in relationship to the whole and the notion that when two identical wholes are divided into the same number of parts, the parts can be shaped differently but still represent the same fractional share. The formal idea and symbolism for fractions are introduced in grade 3.*

3rd Grade

Action Verbs: Understand, recognize draw/sketch, identify, partition, use, explain

Shapes: Two-Dimensional: rhombus, rectangle, square, other 4-sided shapes,
Three-Dimensional: right rectangular prism, right triangular prism, pyramid

Classifications: Quadrilaterals, Squares, Rectangles, Rhombus,

New Vocabulary: (angle was introduced in 2nd grade) right angle, acute angle, obtuse angle, net, right rectangular prism, right triangular prism (rectangular prism was introduced in 1st grade), pyramid, benchmark

Prior Knowledge: In prior grades students partitioned shapes, identified defining versus non-defining attributes, and recognized and drew shapes with teacher given attributes such as number of angles or number of equal faces. A great deal of focus was put on attributes.

Focus of Learning for 3rd Grade: In previous grades students merely recognized rhombuses (rhombi), rectangles and squares, they must now (a) recognize them as quadrilaterals and (b) draw examples of quadrilaterals other than rhombus, rectangle and square. They must also use a right angle as a benchmark for drawing obtuse and acute angles and identify those in rhombi, rectangles and squares. For details see the italicized notes below standard 3.G.1 Also, 3rd grade students build on their previous experience of partitioning shapes into equal parts but they now focus on the fact that while the parts of identical wholes are equal, they need not have the same shape. (See note below 3.G.2 below.)

Geometric concepts in 3rd grade move to developing spatial relations by associating 3-D shapes and the nets that make up the 3-D shapes (right rectangular prism, right triangular prism, and pyramid).

3.G.1 - Understand that shapes in different categories (e.g., rhombus, rectangle, square, and other 4-sided shapes) may share attributes (e.g., 4-sided figures) and the shared attributes can define a larger category (e.g., quadrilateral). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

- *This standard requires students to develop a beginning understanding that shapes can be classified in different ways based on shared attributes/properties. Students are expected to know classifications in two levels – properties that make the shape what it is (see 1.G.1 and 2.G.1) and the overall classification of Quadrilaterals (four sided figures). This includes recognizing examples and non-examples of specific subcategories (rectangles squares and rhombi) of quadrilaterals. In the picture below, an arrow points to examples of four-sided figures (quadrilaterals) that do not belong to the subcategories of rectangle, square or rhombi.*
- *NOTE: Chart 1 below provides more information than is needed at third grade. It is included to see non-examples. To address standards 3.G.1 students need only be able to recognize square, rectangle and rhombus based on the properties that make the shape what is it (see 1.G.1 and 2.G.1 and now include types of angles – 2nd grade was <u>number</u> of angles, now we add <u>types</u> of angles) and understand that because those shapes share the attribute of having 4 sides they belong to a larger category called Quadrilaterals. They also need to understand that there are other 4-sided figures that are quadrilaterals but they are not squares, rectangles or rhombi. Chart 2 below also contains more information than is needed by third grade but is provided for additional information. The information for Trapezoids and Parallelograms will be addressed in 4th grade.*

CHART 1

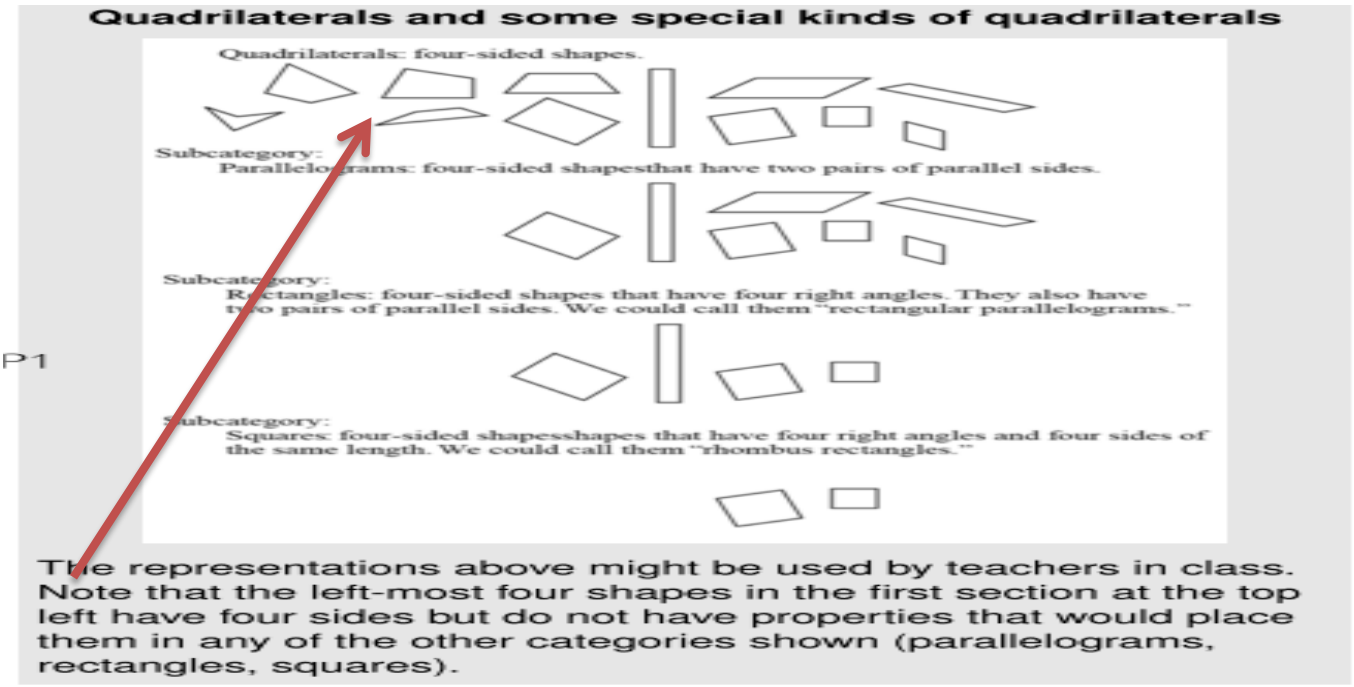
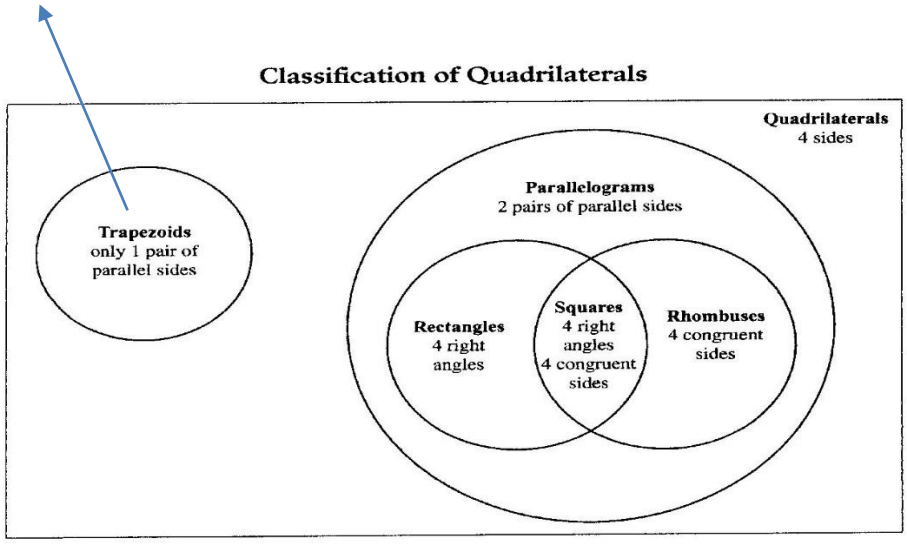


CHART 2

NOTE: Depending on the source used, "trapezoid" can be defined in two ways:

- *A trapezoid is a quadrilateral with exactly one pair of parallel sides.*
- *A trapezoid is a quadrilateral with at least one pair of parallel sides.*
- *According to most sources the inclusive definition (#2) is used more often.*



Property of Quadrilaterals appropriate for 3rd grade

- *Four sides*
- *Right angles*
- *Congruent or non-congruent sides*

3.G.2 - Partition two-dimensional shapes into 2, 3, 4, 6, or 8 parts with equal areas and express the area of each part using the same unit fraction. Recognize that equal parts of identical wholes need not have the same shape.

This standard relates the number of equal parts to the denominators that students will use in 3rd grade fractions. As was the case in 1st grade, the emphasis is on equal parts – a key point when working with fractions. The last portion of the standard emphasizes the fact that while identical wholes, a square for example, can be divided into halves, it can be divided vertically, horizontally diagonally, or in an irregular fashion and the shape is still divided into halves as long as the parts are equal, even though the equal parts do not have the same shape.

3.G.3 - Use a right angle as a benchmark to identify and sketch acute and obtuse angles.
Students were informally introduced to angles in second grade (2.G.1) where they simply counted the number of angles in a shape and drew shapes with a given number of angles – regardless of type of angle. Students will formally measure and draw angles in 4th grade.

3.G.4 - Identify a three-dimensional shape (i.e., right rectangular prism, right triangular prism, pyramid) based on a given two-dimensional net and explain the relationship between the shape and the net.

Also see "Geometry Measurement" 3.MDA.5-6
The emphasis in those standards is on the concept of measurement involving geometric shapes.

4th Grade

Action Verbs: Draw, identify, classify, recognize

Classifications: Quadrilaterals; Parallelograms, Squares, Rectangles, Rhombus, Right Triangles

New Vocabulary: points, lines, line segment, ray, parallel lines, perpendicular lines, symmetry, line symmetry, right triangle

Prior Knowledge: In 3rd grade students put emphasis on classifying shapes in the category of shapes known as quadrilaterals where they understood that shapes such as rhombus, rectangle, square, and other 4-sided shapes belong to the category quadrilaterals but there are shapes other than those that also belong to that category because of defining attributes other than 4-sided figures.

Focus of Learning for 4th Grade: Students build on the category of quadrilaterals by recognizing the impact of parallel or perpendicular lines on shapes in that category. They also now include right triangles as a category. In addition, line symmetry for 2-D figures is introduced.

All though angles were introduced in 3rd grade as the basis for identifying and drawing shapes, the concept of angle is dealt with formally in 4th grade. Students are required to measure and draw angles in whole number degrees, solve addition and subtraction problems to find unknown angles and to understand the relationship of angle measurement to a circle. To understand angles, students should draw points, lines, line segments, rays, parallel and perpendicular lines and identify those in 2-D figures.

4.G.1 - Draw points, lines, line segments, rays, angles (i.e., right, acute, obtuse), and parallel and perpendicular lines. Identify these in two-dimensional figures.

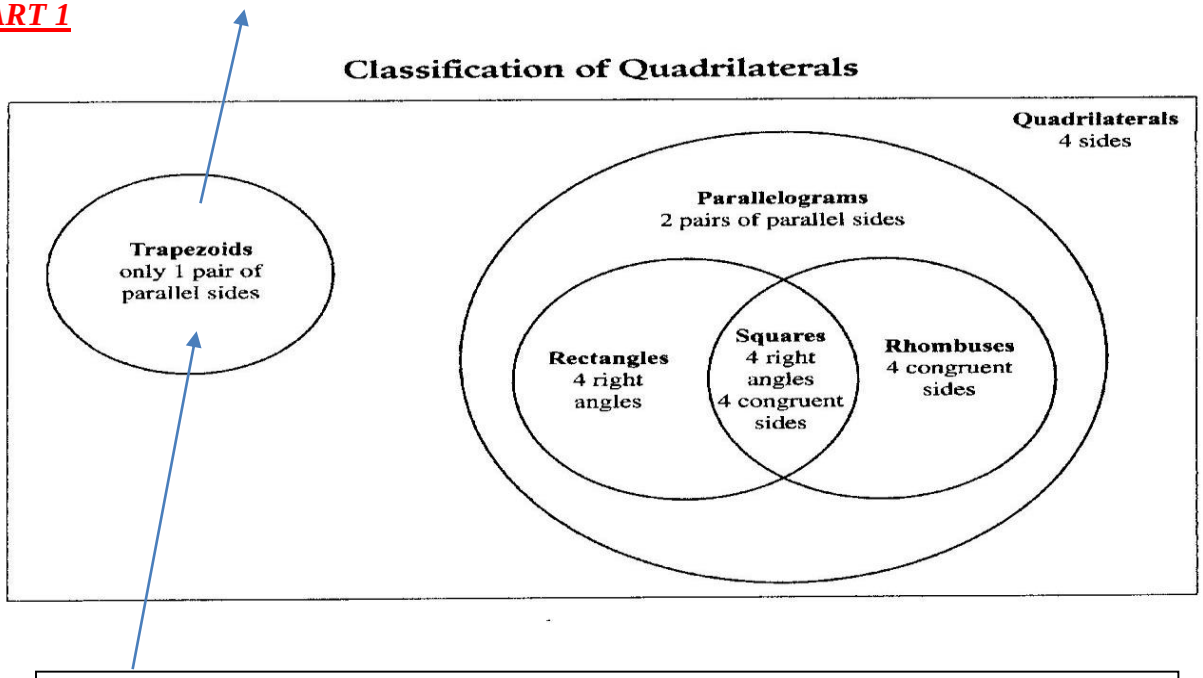
- *Students learn to apply these concepts in varied contexts.*
- *The basic elements of geometry which form its basis are points, lines & planes.*
- *Point:*
  - *As we know that a point is a dot made on a plane by some sharp, pointed object. It may be made on a paper as a hole made with a sharp pin or with the tip of a pen or a pencil on the paper. We represent points by a capital letter. For example, point 'P' or just 'P'. A full stop that we put at the end of each sentence is an example of points. A point, unlike other elements, has no length, no breadth, no thickness.*
- *Line:*
  - *The next element is a line which can be drawn by joining any two points on a plane and extending in both directions. A line is straight and it extends infinitely in both the directions. Given a single point, we can draw an infinite number of lines passing through it. Whereas, there is exactly one line passing through two given points.*
- *Line Segment:*
  - *Line Segment can be defined as a part of a line with two fixed ends. Unlike a line, a line segment has a fixed length. Line segments form the base of the different figures, that we draw in geometry.*

4.G.2 - Classify quadrilaterals based on the presence or absence of parallel or perpendicular lines.

- *See Chart 1 below. This same chart was used for 3rd grade but students did not deal with parallelograms and Trapezoids in 3rd grade.*
- *NOTE: Depending on the source used, "trapezoid" can be defined in two ways:*
  - *A trapezoid is a quadrilateral with exactly one pair of parallel sides.*

- *A trapezoid is a quadrilateral with at least one pair of parallel sides.*
- *According to most sources the inclusive definition (#2) is used more often.*

CHART 1



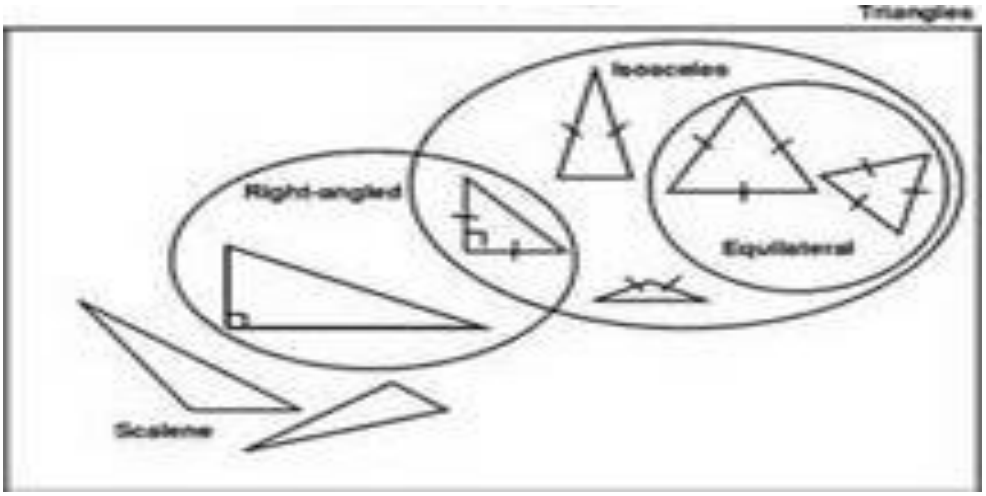
Please see the "NOTE" above Chart 1 regarding the definition for a Trapezoid.

4.G.3 - Recognize right triangles as a category, and identify right triangles.

The term perpendicular lines is introduced in 4.G.2 when working with quadrilaterals. It would be appropriate to include the term when identifying right triangles as well. Chart 2 below contains more information than is required at 4th grade but is provided as the basis to see other types of angle classifications.

CHART 2

Classification of Triangles

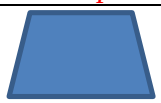


4.G.4 - Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

NOTE: THE FOLLOWING ARE measurement standards necessary for a discussion on geometry.

- MEASUREMENT STANDARDS
  - **4.MDA.5** Understand the relationship of an angle measurement to a circle.
    A circle measures 360 degrees. The four angles of a quadrilateral measure a total of 360 degrees. The three angles of a triangle measure a total of 180 degrees.
  - **4.MDA.6** Measure and draw angles in whole number degrees using a protractor.
  - **4.MDA.7** Solve addition and subtraction problems to find unknown angles in real world and mathematical problems.

Some Specific Kinds of Quadrilaterals

Shape	*Some Properties*	*Name*
	Four sides; at least one pair of parallel sides (NOTE: That is the more widely accepted definition although it is sometimes described as "exactly one pair of parallel sides".	Quadrilateral, Subcategory: Trapezoid – not mentioned in this standard but first addressed in 1st grade.
	Four sides; congruent sides; two pairs of parallel sides (also a parallelogram)	Quadrilateral, Subcategory: Rhombus
	Four sides; two pairs of parallel sides, right angles (also a parallelogram)	Quadrilateral, Subcategory: Rectangle
	Four sides; two pairs of parallel sides, right angles; congruent sides (also a parallelogram)	Quadrilateral, Subcategory: Square NOTE: A square also belongs to the subcategory Rectangle because it has all the properties of a rectangle.

5th Grade

Action Verbs: Graph, define, interpret

New vocabulary: coordinate system, coordinate plane, coordinates, quadrant, horizontal, vertical, plot, graph (as a noun and verb), *x*- and *y*-axes, origin, hierarchy

Classifications: Quadrilaterals, Parallelograms, Squares, Rectangles, Rhombus (same as 4th but with van Hiele level 2 understanding)

Prior Knowledge: In kindergarten (K.G.1), students describe position of objects and begin to develop spatial reasoning by analyzing and comparing 2- and 3-dimensional shapes in different orientations.

Focus of Learning for 5th Grade: The new knowledge in 5th grade geometry is an introduction to graphing in the first quadrant of a coordinate plane. The students are required to graph points on the coordinate plane to solve real world and mathematical problems.

In grades K-4 students dealt with identifying and classifying 2-dimensional shapes. The knowledge gained in each of those grades is combined in 5th grade as students are expected to understand that attributes belonging to a category of 2-dimensional figures also belong to all subcategories (van Hiele level 2 understanding).

5.G.1 Define a coordinate system

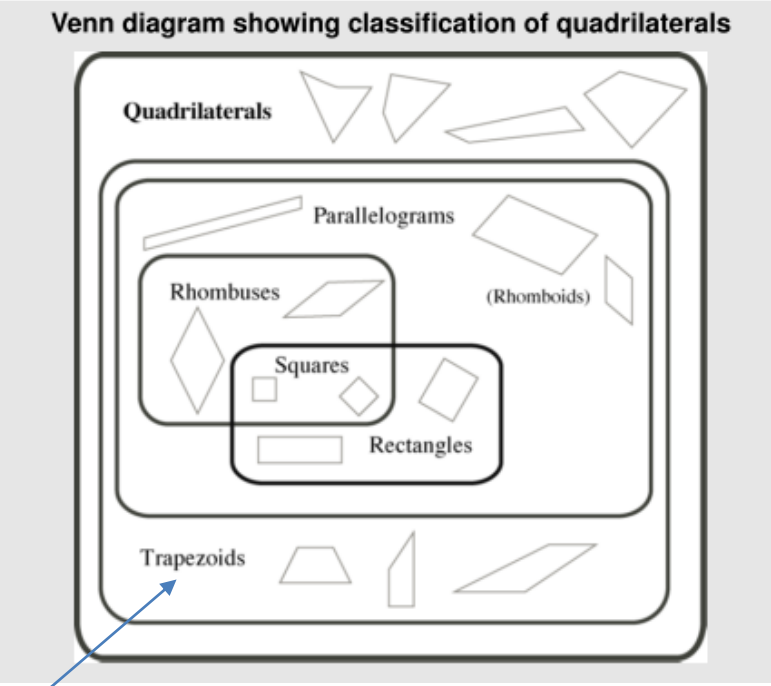
a. The *x*- and *y*- axes are perpendicular number lines that intersect at 0 (the origin);
b. Any point on the coordinate plane can be represented by its coordinates;
c. The first number in an ordered pair is the *x*-coordinate and represents the horizontal distance from the origin;
d. The second number in an ordered pair is the *y*-coordinate and represents the vertical distance from the origin.

5.G.2 - Plot and interpret points in the first quadrant of the coordinate plane to represent real-world and mathematical situations.

5.G.3 - Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category.

- *Students learn to analyze and relate categories of 2D shapes explicitly based on their properties. Based on analysis of properties they classify 2D figures in hierarchies (5.G.4). For example, they conclude that all rectangles are parallelograms, because they are all quadrilaterals with two pairs of opposite, parallel, equal-length sides. In this way, they relate certain categories of shapes as subclasses of other categories. This leads to understanding propagation of properties; for example, students understand that squares possess all properties of rhombuses and of rectangles.*

5.G.4 - Classify two-dimensional figures in a hierarchy based on their attributes.



NOTE: Depending on the source used, "trapezoid" can be defined in two ways:

- *A trapezoid is a quadrilateral with exactly one pair of parallel sides.*
- *A trapezoid is a quadrilateral with at least one pair of parallel sides.*
- *According to most sources the inclusive definition (#2) is used more often.*

- *In 3rd grade students were expected to know classifications in two levels – properties that make the shape what it is and the overall classification of Quadrilaterals (four sided figures). That included recognizing examples and non-examples of specific subcategories of quadrilaterals. In 4th grade the attribute of parallel and perpendicular lines was included when classifying shapes. In 5th grade students bring all that knowledge together to examine how subcategories of quadrilaterals can "overlap" because of the properties they share.*
  - *For example, see squares in the Venn diagram above – a square belongs to the major category "Quadrilaterals" (has four sides) and to the subcategories Rhombus (congruent/equal side lengths), to the subcategory Rectangle (right angles). However, rhombi are not squares because rhombi do not have right angles and rectangles are not squares because rectangles do not have congruent/equal side lengths.*
  - *For example, see parallelograms in the Venn diagram above – all parallelograms are quadrilaterals because they have four sides but not all quadrilaterals are parallelograms because not all quadrilaterals have two pairs of parallel sides.*

Required Properties of Quadrilaterals

- *Four sides*
- *Four angles*

Required Properties of Parallelogram

- *opposite sides parallel*

- *opposite angles have the same measure (congruent)*
- *opposite sides the same/congruent*

Required Properties of Rectangle

- *opposite sides parallel*
- *opposite angles the same/congruent*
- *opposite sides the same/congruent*
- *all angles right*

Required Properties of Square

- *opposite sides parallel*
- *opposite angles the same/congruent*
- *opposite sides the same/congruent*
- *all angles right*
- *all sides the same/congruent*

Required Properties of Rhombi

- *opposite sides parallel*
- *opposite angles the same/congruent*
- *opposite sides the same/congruent*
- *all sides the same/congruent*

Clarifying comments added by Mary L. Ruzga, Office of Standards and Learning